ZENO OF ELEA, LOCOMOTION, INFINITY, AND TIME

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1. GENERALITIES


2. PHILOLOGY

kinesis is “motion”, “movement”, “change”. But before Aristotle kinesis means more locomotion or disturbance than general change.
Snell [207, p. 217, chapter 9], [207, pp. 241–244, chapter 10]

3. MATHEMATICS

Heath [94, pp. 271–283]
Szabó [220]

4. PYTHAGOREANS

Guthrie [86]
Horky [100]
Archytas of Tarentum on geometric proportion [101]
Philolaus of Croton [102]
Burkert [23, pp. 285–288]

5. XENOPHANES

Testimonia on Fragment 26 from the peripatetic On Melissus, Xenophanes, and Gorgias [124, pp. 204–210]

6. HERACLITUS

Heraclitus saying things are and are not is like saying that things are specified by their position and velocity, not just position. (A photo does not tell you everything about an object.)

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7. Parmenides and Melissus

The testimonia, A section of Diels and Kranz, on Parmenides are translated in Gallop [73].
Mourelatos [156, pp. 118–119]
Parmenides criticizes motion in B8.26–33, and Melissus in B7.7–10.
Melissus 4: “Nothing that has a beginning and an end is either everlasting or infinite.” [66, p. 48], DK30B4.
Palmer [173]

8. Zeno

Zeno [123, pp. 45, 67, 71–85]
Waterfield [247]
Four fragments arguing that if Things are Many a contradiction follows [66, p. 47], DK29B.
Algra [4]
Guthrie [87]
Solmsen [209, p. 18]
Lloyd [131]

9. Anaxagoras

Fragment 3: “Nor of the small is there a smallest, but always a smaller (for what-is cannot not be) – but also of the large there is always a larger. And [the large] is equal to the small in extent, but in relation to itself each thing is both large and small.” Curd [47, pp. 38–42]
Schofield [193, Chapter 3]

10. Democritus

Democritus DK68B155: “If a cone were cut with a plane parallel to the base, …” [66, p. 106]
Lura [136]
Fragments in Taylor [223]

11. Empedocles

12. Protagoras

Protagoras of Abdera DK80B7: tangents to circles [66, p. 126].

13. Gorgias

Gorgias of Leontini DK82B3: nothing exists, sophisticated argument [66, p. 128]

14. Antiphon the Sophist

Infinite divisibility. Fragments 1, 13 [175]
15. Diogenes of Apollonia

16. Plato

Friedländer [67]
Taylor [222]
Cornford Parmenides [40]
Cornford Timaeus [39]

Plato refers to Zeno making his audience think that things are one and many and at rest and at motion in Phaedrus 261d and Parmenides 129e.

Plato’s Parmenides [5, pp. 93–98, 250–260]
Cornford [41, p. 160] refers to Timaeus 39b, and writes “Distance in space is measurable psychologically, by expenditure of strength; but time-distance can be measured only by counting the rhythmical repetition of the same occurrence.”

Cornford [41, p. 131]: the soul can move itself. Laws 896a. Cornford cites Aetius and Sextus Empiricus.

17. Aristotle

Peripatetic On indivisible lines and On Melissus, Xenophanes, and Gorgias [98]
Heath [96]
Ross [185, p. 94]
Roark [183]
Bolotin [17]
Cherniss [31]

Aristotle De anima, Polansky [176, p. 96]
Aristotle Physics 239b9–14.
Aristotle Prior Analytics 65b16–21.

18. Aristoxenus fl. 335 BC

Levin [126] and Barker [10] infinite divisibility in music theory

19. Heraclides Ponticus c. 390 BC–c. 310 BC

Sharples [203]

20. Xenocrates c. 396/395 – c. 314/313 BC

Sambursky [189, p. 91]
Dillon [55, pp. 111ff]: indivisible lines.

21. Theophrastus of Eresus c. 371 BC – c. 287 BC

, Sharples [201]

22. Praxiphanes fl. c. 300 BC

Sharples [205]

23. Strato of Lampsacus c. 335 BC – c. 269 BC

Sharples [204]
24. Eudemus of Rhodes c. 370 BC – c. 300 BC

Sharples [202]
Wehlri fragments 37, 60, 62 and 78 [249].

25. Diodorus Cronus died c. 284 BC

Gaskin [74, pp. 60, 64, 108, 252, 260]

26. Archimedes c. 287 BC – c. 212 BC

Heath [93]
Dijksterhuis [52]

27. Plutarch c. 46 – c. 120

Moralia, book XIII, De communibus notitiis adversus Stoicos [32]

28. Epicurus 341 BC – 270 BC

Letter to Herodotus 57, 61–62. In 38, Epicurus says that there must be a void lest things not move.
Vlastos [240]
Milton [152]

29. Chrysippus c. 279 BC – c. 206 BC

Gould [80, pp. 112–119, chapter V, §1f]
Bobzien [16]

30. Polybius c. 200 BC – c. 118 BC

Histories, Book IV, Chapter 40: “For given infinite time and basins that are limited in volume, it follows that they will eventually be filled, even if silt barely trickles in. After all, it is a natural law that, if a finite quantity goes on and on increasing or decreasing – even if, let us suppose, the amounts involved are tiny – the process will necessarily come to an end at some point within the infinite extent of time.”

31. Asclepiades of Bithynia c. 129/124 BC – 40 BC

Vallance [237]

32. Antiochus of Ascalon c. 125 BC – c. 68 BC

Dillon [54, p. 82]: “there exists nothing whatever in the nature of things that is an absolute least, incapable of division.” Acad. Post. 27ff.

33. Varro 116 BC – 27 BC

Sedley [196]

34. Cicero 106 BC – 43 BC

De natura deorum, I.xxiii.55: no such thing as an indivisible body
Dillon [55, p. 170]: matter is “capable of infinite section and division”.
Academica, I.vii.27: matter and space are infinitely divisible
35. Posidonius c. 135 BC – c. 51 BC

Fragment 98 [109, pp. 395–403]

36. Lucretius c. 99 BC – c. 55 BC

Lucretius *De rerum natura* 2.238–2.239 [135].

37. Philo of Alexandria c. 25 BC – c. 50

Goodenough [78, pp. 127–139]

38. Alexander of Aphrodisias fl. c. 200

Quaestiones 1.21 and 1.22 [199, pp. 74–75] and 3.12 [200, p. 67]

Ancient Commentators on Aristotle


40. Sextus Empiricus c. 160–210

Against the Physicists [12]

Adversus mathematicos I: Against the grammarians [15, pp. 8, 61, 166]

Outlines of Scepticism [6]

Hankinson [89] on moments of time, and on bodies and surfaces in space.

41. Numenius of Apamea fl. c. 150

Numenius [85, p. 58]: “Bodies, containing nothing unchangeable, are naturally subject to change, to dissolution, and to infinite divisions.”

42. Plotinus c. 204/205–270

43. Dionysius of Alexandria c. 200 – 264

Cleve [35]

44. Porphyry 234–c. 305

Gaiser [70, p. 482]. Simplicius, *In Physicorum*, 454, 6, quotes Porphyry. Take a definite length one cubit long. Divide it in half. Leave one-half undivided and divide the other again. If we continue dividing, Porphyry says that “there is a certain infinite nature enclosed in the cubit, or rather several infinities, one proceeding to the great and one to the small.” The infinitely large is the increasing number of segments.

45. Iamblichus c. 245–c. 325

The Theology of Arithmetic, “On the Dyad” [246, p. 45]: “length is both infinitely divisible and infinitely extensible.”
46. EUSEBIUS 260/265–339/340

*Præparatio evangelica*, book XV, chapter XXII: “But in fact the whole sentient is one: for how could it be divided? For there can be no correspondence of equal to equal, because the ruling faculty cannot be equal to each and every sensible object. Into how many parts then shall the division be made? Or shall it be divided into as many parts as the number of varieties in the object of sense that enters? And so then each of those parts of the soul will also perceive by its subdivisions, or the parts of the subdivisions will have no perception; but that is impossible. And if any part perceive all the object, since magnitude by its nature is infinitely divisible, the result will be that each man will also have infinite sensations for each sensible object, infinite images, as it were, of the same thing in our ruling faculty.”

47. EPHREM THE SYRIAN C. 306–373

Possekel [178]

48. CALCIDIUS FL. C. 400

van Winden [239, p. 155]

49. SYRIANUS DIED C. 437

Wear [248]
Syrianus [53, p. 57]

50. PROCLUS 412–485

Opsomer [171] on the *Elements of Physics*
Morrow and Dillon [155]
*Elements of Theology* [56]

51. IRENAEUS OF LYONS C. 130–C. 202

*Adversus haereses*, II.1.4: “These remarks are, in like manner, applicable against the followers of Marcion. For his two gods will also be contained and circumscribed by an immense interval which separates them from one another. But then there is a necessity to suppose a multitude of gods separated by an immense distance from each other on every side, beginning with one another, and ending in one another.”

52. CLEMENT OF ALEXANDRIA C. 150–C. 215

*Stromata*

53. HIPPOLYTUS OF ROME 170–235

*Refutation of All Heresies*, IV.51.

54. ORIGEN 184/185–253/254

*De Principiis*

55. ALEXANDER OF LYCOPOLIS FL. EARLY FOURTH CENTURY

Infinite divisibility of matter [238]
56. Hilary of Poitiers c. 300–c. 368

*De Trinitate.* Meijering [149]

57. Basil the Great 329/330–379

*Hexaemeron*, Homily I, article 4: “These men who measure the distances of the stars and describe them, both those of the North, always shining brilliantly in our view, and those of the southern pole visible to the inhabitants of the South, but unknown to us; who divide the Northern zone and the circle of the Zodiac into an infinity of parts, who observe with exactitude the course of the stars, their fixed places, their declensions, their return and the time that each takes to make its revolution; these men, I say, have discovered all except one thing: the fact that God is the Creator of the universe, and the just Judge who rewards all the actions of life according to their merit.”

*Hexaemeron*, Homily I, article 6: “The beginning, in effect, is indivisible and instantaneous. The beginning of the road is not yet the road, and that of the house is not yet the house; so the beginning of time is not yet time and not even the least particle of it. If some objector tell us that the beginning is a time, he ought then, as he knows well, to submit it to the division of time – a beginning, a middle and an end. Now it is ridiculous to imagine a beginning of a beginning. Further, if we divide the beginning into two, we make two instead of one, or rather make several, we really make an infinity, for all that which is divided is divisible to the infinite.”

58. Gregory of Nyssa c. 335–c. 395

*Against Eunomius*, Book I. See entry for infinity in [142].

59. Augustine 354–430

Letter 3 (to Nebridius), article 3.
O’Daly [169, p. 157].
Knuutila [113]

*De Trinitate*, XI, article 17 and XV, chapter 12. In [144]

*Confessions*, book XI [9].

60. Themistius 317–c. 390

_in Physicorum_ 91.29–30.

61. Simplicius

*In Physicorum* 139.27–140.6, Zeno’s arguments against plurality. See Curd [46, pp. 171–186]

Simplicius [236]

62. John Philoponus c. 490–c. 570

63. Olympiodorus

Furley [69]
64. KALAM

The kalam cosmological argument, in Craig and Sinclair [44]
Zimmerman [259]
Wolfson [257]

65. AN-NAZZAM c. 775–c. 846

Ibrahim An-Nazzam

66. AL-KINDI c. 801–c. 873

Al-Kindi [3]

67. IBN QURRA c. 826–901

Rashed [180]

68. ALFARABI c. 872–950/951

Alfarabi [137, pp. 101–111]

69. AL-SIJZI c. 945–c. 1020

Rashed [181]

70. AL-BIRUNI 973–1048

Letter to Avicenna: “If the sun is west of the moon in the sky, with a definite space between them, then even though the moon moves much faster than the sun, it should never be able to catch it. For the space between them can be conceived as divisible into an infinite number of parts; but how can a body moving with a finite speed cross an infinite number of spaces?” [198, p. 820]

71. AVICENNA c. 980–1037

McGinnis [145]
Rashed [179]

72. SAINT ANSELM OF CANTERBURY c. 1033 – 1109

, On the Incarnation of the Word, §15.

73. AL-GHAZALI c. 1058–1111

Goodman [79]

74. AVEMPACE c. 1085–1138

Lettinck [125]

75. AVERROES 1126–1198

Glasner [75]
Goldstein [77]
76. JEWISH PHILOSOPHERS

Saadia Gaon [184]
Maimonides, Guide for the Perplexed, I.73 106a
Hasdai Crescas, in [91] and Wolfson [256]
Rudavsky [187]

77. GERSHONIDES 1288–1344

Gersonides [186]
Kohler [115]

78. ROYAL MS 4 A XIV, 12TH CENTURY

Royal MS 4 A XIV, Against wens, ll. 11–13, Storms [214, p. 155, no. 4]: “May you become as small as a linseed grain,/ and much smaller than the hipbone of an itchmite,/ and may you become so small that you become nothing.”

79. PETER ABELARD 1079–1142

King [110, p. 94]

80. HUGH OF SAINT VICTOR c. 1096–1141

Didascalion, chapter 17: “From this consideration derives the axiom that continuous quantity is divisible into an infinite number of parts, and discrete quantity multipliable into a product of infinite size. For such is the vigor of the reason that it divides every length into lengths and every breadth into breadths, and the like – and that, to this same reason, a continuity lacking interruption continues forever.” [83, p. 58]

81. JOHN OF SALISBURY

Metalogicon

82. WILLIAM OF CONCHES c. 1090–c. 1154

83. HERMAN OF CARINTHIA c. 1100–c. 1160

De Essentiis [25, p. 252]

84. WILLIAM OF AUVERGNE 1180/1190–1249

William also presents arguments that the view that a continuum, such as time, is infinite results in paradoxes (OO I, 698a-700b).

William had read Aristotle’s Physics and agrees with Aristotle that time and motion are coextensive (OO I, 700a). Yet he does not propose Aristotle’s definition of time as the number of motion in respect of before and after. Rather, in his account of the essential nature of time he describes time simply as being that flows and does not last, “that is, it has nothing of itself that lasts in act or potency” (OO I, 683a; Teske [224, p. 102]), De universo.

85. RICHARD RUFUS DIED C. 1260

Lewis [128]
86. Peter of Spain c. 1215–1277

*Syncategoreumata* [51], chapters 5 and 6

87. Roger Bacon c. 1214–c. 1292

Roger Bacon [83, p. 396]

In his *Opus tertium, Opera quaedam hactenus inedita*, cap 39, pp. 134–135, Roger Bacon writes:

> A body’s potential for division cannot be reduced to actuality, purely and completely. It is a potentiality that one can only reduce to actuality impurely and incompletely, where there is always a mixture with a potential for further actualization; it is always reduced but in such a way that there remains the potential for another division. That is the potential of the continuum and that which constitutes infinite divisibility: when this potential is reduced by actual division, the possibility of another division is not excluded. Actually, it is required; in fact, the portion which is the result of division is a magnitude; hence it is still divisible, and so forth to infinity.

88. Robert Grosseteste c. 1175–1253

Lewis [127] and [128]

89. Albertus Magnus c. 1200–1280

Twetten, Baldner, and Snyder [235]

Fox [64]

Money as infinitely divisible quantity [108]

90. Thomas Aquinas 1225–1274

*Summa Theologica*, prima pars, q. 7, article 3; q. 48, article 4; q. 53, article 2.

*Commentaria in octo libros Physicorum*, articles 69, 377–379 [7, pp. 188–189]

*In libros De generatione et corruptione expositio*, lecture 7, article 56; lecture 4, article 29.

91. Arnald Villanova c. 1240–1311

McVaugh on minima natura [148, p. 97]

92. Saint Bonaventure 1221–1274

93. Henry of Ghent c. 1217–1293

Brower-Toland [21]

94. Peter John Olivi

Pasnau [174]

95. Ramon Llull

Lohr [132]
96. Duns Scotus
Trifogli [231]

97. Godfrey of Fontaines
Dales [49, pp. 185–186, 202–203, 233, 255]

98. Henry of Harclay
Murdoch [161]
Dales [48]

99. Thomas Bradwardine
Dolnikowski [57]

100. Johannes de Muris
Busard [27, p. 35]. Porism to Prop. 19: “the horn-like angle is infinitely divisible by circular lines, can increase infinitely by diminishing the circles, and can decrease by augmenting the circles.”

101. Walter Chatton
Murdoch and Synan [165]

102. Gerardus Odonis c. 1285–1349

103. Nicolas Bonet

104. Nicholas of Autrecourt c. 1299–1369
The universal treatise.

105. Robert Kilwardby
Trifogli [232]

106. William of Auxerre
Tummers [234]

107. Peter of Auvergne
Galle [72, pp. 277*–330*]

108. John Buridan
Murdoch and Thijssen [166]

Buridan gives an example in his Quaestiones super octo libros Physicorum, lib. III, quaest. XVIII, fol. 63, col. d, about a cylindrical column divided into proportional parts [61, p. 58]. In the same work, cols. c, d, Buridan writes “Assuredly, when I take my book, I take an infinity of parts of my book, for I am taking three parts, 100 parts, 1000 parts, and so forth without end. But what is impossible, is that one takes an infinity of parts successively, counting one after the other.”
109. Robert Holcot

A man is alternately meritorious and sinful in proportional parts of the last hour of his life. This suggests the geometric series

\[ \sum_{n=0}^{\infty} (-r)^n. \]


110. Aegidius Romanus

Porro [177]
Trifogli [229]

111. William Crathorn

112. Peter Auriol c. 1280–1322

113. William of Ockham

Goddu [76]
Murdoch [162]

114. John Bassolis

John Bassolis in his *In Quatuor Sententiarum libros. Quaestiones in Primum Sententiarum*, dist. XLIII, quaest. unica, fol. 213, col. c [61, p. 99]

The division of any finite quantity into parts whose magnitudes follow a constant relationship can be pursued to infinity. It is the same with the increase of a quantity by the addition of similar divisible parts. Divine virtue itself cannot reduce this division or this increase to actuality *in facto esse*, but only *in fieri*, and this is because the reality or nature of things repulses this actualization. But this in no way constitutes an objection to our proposition.

115. Richard of Middleton

Richard of Middleton in his commentary on Lombard’s *Sentences Super quattuor libros Sententiarum Petri Lombardi quaestiones subtilissimae*, lib. I, dist. XLIII, art. 1, quaest. IV, vol. 1, p. 386, col. b [61, p. 79]

When one states that any continuum is divisible to infinity, I reply that it is true as long as one understands it thus: It can be divided without end, but in such a way that the number of parts already obtained is always finite. If one admits that it is thus divided, no impossibility results; the existence of an infinite *in facto esse* does not result, only the existence of an infinite *in fieri* which one commonly calls an infinite in actuality mixed with potentiality.

116. William Heytesbury

Wilson [254]
Longeway [134]
117. Richard Kilvington

Kretzmann and Kretzmann [120]
Jung and Podkoński [107]

118. John Dumbleton

119. Walter Burley

Duhem [61, p. 57] quotes from Walter Burley’s *Super octo libros physicorum* [219], lib. III, tract. II, cap. 4, fol. 70, col. b:

What we have just expounded upon proves the truth of the following proposition which is not known by many: Given any line, one can mark off segments whose lengths decrease proportionally, and one can also indicate a point which cannot be reached by a finite operation. That will occur if one takes as the first segment half the length to the extremity which cannot be reached by a finite operation; one takes as the second segment half the first segment, and so forth. On the other hand, every point before the extremity can be reached by a finite operation. That can easily be demonstrated geometrically, but for now we will not insist on its demonstration.

Spade [24, pp. 74–75, 117–123]

120. Albert of Saxony

Sarnowsky [192]
Biard [13]

121. Walter Odington

122. Richard Swineshead

123. Nicole Oresme

*Questiones super Physicam* [29]

124. Gregory of Rimini

Cross [45]
Thakkar [225]

Gregory of Rimini [228, p. 441], in the first conclusion of the first article on the first book of his commentary on the *Sentences*, says that “God can make any actually infinite multitude”, and gives the example of making an infinite number of angels in an hour, and talks about this using proportional parts: in each proportional part of an hour, God creates and preserves an angel, and at the end of the hour there are infinitely many angels. Rimini also talks about God creating an infinite magnitude [228, p. 445]. Also, creating infinity charity [228, pp. 446–447].

Gregory of Rimini in his *In primum Sententiarum*, dists. XLII, XLI, XLIV, quaest. IV, art. 2, fol. 190, col. c (fol. 175, col. a) [61, pp. 115–116]:

When one says, infinity is something never completed, I reply that it is so if its infinitely numerous parts are acquired in equal durations; if, for example, each part of this infinity were acquired after an hour or a moment, or some other determined quantity of time.
In that case, it would have to be that the time would have an infinity of equal parts and, consequently, that it would be infinite. Since, in any case, it is impossible that an infinite time whose first part is given becomes a past time, an infinity could not be totally completed or surpassed in this manner.

One says, infinity is something such that when one takes any part of it whatever, there always remains another part to be taken. I reply that this proposition must be understood as the previous one, by admitting that the parts taken successively are all of the same magnitude and that they are all taken in equal times. If one takes, in some time, a portion of infinity, then in a time equal to the one in which the first part was taken, one takes an equal portion, and one continues in this fashion, there will always remain something to be taken of this infinity, and it will never be taken in its totality....

But once equal parts of the infinity are not taken in equal times, but in times whose durations decrease in geometric progression... there is no longer any inconsistency in the infinity being taken in its totality, as long as there is no obstacle of some other nature to this. Similarly, there is no inconsistency in that the infinite multitude of parts of time, in which the successive parts of the infinite are taken, come to be completely past, as we have already stated. Not only is there no inconsistency in this, but it is necessary that it be.

Gregory of Rimini, responding to Zeno’s paradoxes as stated by writers such as Henricus Hibernicus, Adam Goddam, and Clienton Lengley, in his In secundum Sententiarum, dist. II, quaest. II, art. 1, fol. 34, col. c:[61, p. 57] “In any magnitude there is an infinity of proportional parts, infinity being taken syncategorematically; a result of this is that none of these parts is the last one.”

Duhem writes [61, p. 126] “it seems however that Oresme speaks the language of a disciple of Gregory of Rimini, of a defender of the categorematic infinite”. In his Trait é du Ciel et du Monde, livre I, fol. 11, col. d (pp. 109–11), Nicole Oresme responds to Aristotle’s statement in De Caleo that an infinite body would have to be infinitely heavy: [61, p. 127]

But it seems to me that the reason given above is not evident without adding another assumption. For, in accordance with the second reply, I assume a body to be infinite, and I take or assign in this body a finite portion, spherical in shape, called A. Next I take another sphere B from the same section, and of the same shape, and then another sphere C, exactly like A and B, proceeding in this manner without stopping. In this way, it appears that there are, in this infinite body, infinite equal parts A, B, C, D, and so on without limit.

Now I posit that in the portion called A there should be distributed the weight of one half-pound, and in B there should likewise be distributed one-half of another half-pound, and in C one-half of the residue of a pound, and in D half of this remainder, which would be one-sixteenth part of a pound, and so on without end.
It appears then that the entire infinite body will weigh only one pound, while $A$ will weigh as much as all the other portions, however many, taken together.

125. ADAM DE WODEHAM
Wood [258]
Courtenay [42]

126. MARSILIUS OF INGHEN
Hoenen [99]

127. BLASIUS OF PARMA
Biard [14]

128. JEAN DE RIPA

129. JOHN WYCLIFFE C. 1331–1384

130. PATIENCE
Eldredge [62]

131. JOHN DEE
Clulee [36]

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