

# Turning a rectangle into a square in the Sulbasutras

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*Vedic Index of Names and Subjects*, Macdonell and Keith:

rajju, rope

*śaṅku*: “Śaṅku in the Rigveda and later denotes a ‘wooden peg.’ ”

Katyayana Sulbasutra [33]

9, 1874-75: 292-298; 10, 1875-76: 17-22, 44-50, 72-74, 139-146, 166-170,  
186-194, 209-218; NS 1, 1876-77: 316-22, 556-578, 626-642, 692-706, 761-770.

Baudhāyana Śulbasūtra 1.4 [51, p. 77]:

Having desired (to construct) a square, one is to take a cord of length equal to the (side of the) given square, make ties at both ends and mark it at its middle. The (east-west) line (equal to the cord) is drawn and a pole is fixed at its middle. The two ties (of the cord) are fixed in it (pole) and a circle is drawn with the mark (in the middle of the cord). Two poles are fixed at both ends of the diameter (east-west line). With one tie fastened to the eastern (pole), a circle is drawn with the other. A similar (circle) about the western (pole). The second diameter is obtained from the points of intersection of these two (circles); two poles are fixed at two ends of the diameter (thus obtained). With two ties fastened to the eastern (pole) a circle is drawn with the mark. The same (is to be done) with respect to the southern, the western and the northern (pole). The end points of intersection of these (four circles) produce the (required) square.

Baudhāyana Śulbasūtra 1.5 [51, p. 77]:

Now another (method). Ties are made at both ends of a cord twice the measure and a mark is given at the middle. This (halving of the cord) is for the east-west line (that is, the side of the required square). In the other half (cord) at a point shorter by one-fourth, a mark is given; this is the *nyañcana* (mark). (Then) a mark is given

at the middle (of the same half cord) for purposes (of fixing) the corner (of the square). With the two ties fastened to the two ends of the east-west line (*pr̥ṣṭhyā*), the cord is to be stretched towards the south by the *nyañcana* (mark); the middle mark (of the half cord) determines the western and the eastern corners (of the square).

Katyayana Śulbasūtra 1.1 [58, p. 95], Thibaut:

We will explain the employment of the cords (by means of which the sacrificial areas are measured out).

Katyayana Śulbasūtra 1.2 [58, p. 95], Thibaut:

Having, on a level spot, planted a pole and having described (round it) a circle by means of a cord fastened to the pole (or else “measured by the pole”) one fixes a pole on each of the two points where the end of the pole’s shadow touches the two halves of the circle; this (the line connecting these two points) is the east-line.

Katyayana Śulbasūtra 1.2 [51, p. 120], Sen and Bag:

Having put a pole on a level ground and described a circle round it by means of a cord (fastened to the pole), a pole is fixed on each of the two points where the end of the pole’s shadow touches (the two halves of the circle). This (line joining the two points) is the east-west line (*prācī*). Then after doubling (a given) cord, two loops (made at its two ends) are fixed at the two poles (of the *prācī*), and (the cord is stretched towards south by its middle point where) a pole is fixed; the same is repeated to the north. This (line joining the two poles) is the north-south line (*udīcī*).

Katyayana Śulbasūtra 1.3 [58, p. 99], Thibaut:

Having by a cord doubled their interval (i.e. having taken a cord whose length is double the interval of the two poles) and having made slings at its two ends and fastened them to the two poles one stretches the cord towards the south and plants a pole at its middle, the same is done on the north side, this is the north-line.

Katyayana Śulbasūtra 1.3 [51, p. 120], Sen and Bag:

Two loops are fixed at the two ends of a cord. Marks are (to be given) at the *śronīs*, the *aṃsas*, the *nirañchana* and the *samāsabhaṅgas*. A pole is fixed at each end of the east-west line (of desired length); likewise (a pole is fixed at each of) the two *śronīs* (west corners) and the two *aṃsas* (east corners). Having fixed the loops at the two poles (on the east-west line), the cord is to be stretched by the *nirañchana* mark towards the south-east corner. The same is done towards the north-east corner. After interchanging (the loops of the cord on the poles), the same is repeated. This is the method (of construction of squares and rectangles) in all cases.

Katyayana Śulbasūtra 1.3 [58, p. 99], Thibaut:

Katyayana Śulbasūtra 1.3 [58, p. 99], Thibaut:

Katyayana Śulbasūtra 1.3 [58, p. 99], Thibaut:

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