

NICOLE ORESME

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Courtenay on Oresme's early career [8]. Clagett biography of Oresme [7]. Grant [11, pp. 106–107] gives the following scheme for medieval *questiones*:

1. The statement of the question;
2. principal arguments (*rationes principales*), usually representing alternatives opposed to the author's position;
3. opposite opinion (*oppositum*, or *sed contra*), a version of which the author will defend. In support of this opinion, the author often cites a major authority, often Aristotle himself; or cites from a commentary on a work of Aristotle; or invokes a theological authority in a theological treatise, such as a *Commentary on the Sentences of Peter Lombard*;
4. qualifications, or doubts, about the question, or about some of its terms [optional];
5. body of arguments (author's opinions by way of a sequence of conclusions);
6. brief response to refute each principal argument.

Kenny and Pinborg [14]:

From about 1260 we find commentaries consisting only of a series of *quaestiones*, each of which has the basic form of a disputation. Presumably this reflects the development of a new kind of lecture. We do not know to what extent, if any, such *quaestiones* were staged as real disputations. Certainly from the fourteenth century onwards we have testimony that they were only read aloud by the master. (pp. 20–21)

Perhaps the *disputatio* simply grew out of the other and older vehicle of professorial instruction: the *lectio*, or lecture. In the course of expounding a text a commentator, from time to time, is bound to encounter difficult passages which set special problems and need extended discussion. When we are dealing with a sacred or authoritative text, the difficult passages will have given rise to conflicting interpretations by different commentators, and the expositor's duty will be to set out and resolve the disagreements of previous authorities. Thus the *quaestio* arises naturally in the course of the *lectio*, and the disputation and the lecture are the institutionalised counterparts of these two facets of a method of study oriented to the interpretation of texts and the preservation of tradition. (p. 25)

Kenny and Pinborg [14, pp. 29–30] write about *quaestiones*:

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Most medieval philosophical literature reflects teaching practice and its form; even writings that were never delivered as lectures or held as disputations assume the traditional forms.

See also Lawn [16] on *quaestiones*.

Accordingly, a large proportion of medieval philosophical literature is in the form of commentaries...

Besides the literal commentaries there are commentaries in the form of a series of questions. Originally such questions formed only the latter part of lectures, but apparently they gradually became independent from the traditional lecture-form. From the latter half of the thirteenth century we find such commentaries, often consisting only of questions; sometimes short paraphrases of passages of the *littera*, otherwise neglected, are still given; but often only the opening words are left from the old structure.

They describe the scheme of *quaestiones* [14, pp. 30–32]:

Such questions retain the simplest possible structure of a disputation. First a problem is stated in the *titulus quaestionis* which is always formed as a question introduced by ‘*utrum*’...

After the *titulus quaestionis* follows a short series of principal arguments for one of the two possible answers to the problem stated, frequently introduced by a formula such as ‘*et arguitur (videtur) quod sic/non*’. These arguments normally defend the position eventually refuted. The normal number of arguments is two or three. Then follow arguments on the opposite side of the issue. They are often fewer in number (often only one), and are frequently nothing but references to authority. This is justifiable according to medieval tradition, even if an argument from authority was held to be the weakest form of argument. Real arguments were normally given in the solution of the question, where the author adopts the position he himself means to defend.

The solution (or *corpus quaestionis*) is introduced by phrases such as ‘*ad hoc dicendum/dico*’ and states the conclusions of the author, accompanied by some arguments and distinctions necessary to carry through the solution. These arguments are normally more carefully organised and articulated, but still may take as their major premisses propositions which have not been or are not proved ‘demonstratively’ but are only regarded as generally acceptable. Frequently several previous opinions on the subject are summarised and refuted, before the author states his own opinion...

The last part of a question contains the refutations of the arguments leading to the solution opposite to the one advocated by the author. They often contain some distinctions which were thought not to be necessary to the general solution of the problem but of importance only to solving one of the counter-arguments.

Murdoch [22, pp. 567–568]:

Almost all the scholastics followed Aristotle’s distinction of permissible from non-permissible infinites, formulating a variety of alternative ways of expressing this distinction. The most popular of

these alternative expressions was the claim that the rejected actual infinite was a quantity so great that it could not be greater (*tantum quod non maius*), while the permissible potential infinite was a quantity that was not so great but that it could be greater (*non tantum quin maius*). The scholastics themselves often pointed out that the latter was really only an indefinite finite, as was made explicit in any number of ‘expositions’ of propositions involving this type of infinite.

Clagett, *Archimedes in the Middle Ages*, volume III, parts I and II

Johannes de Tinemue’s redaction of Euclid’s *Elements*, p. 35

Oresme’s *Questiones super geometriam Euclidis* [4] (the first edition was reviewed in detail by Murdoch [19, p. 69]) are 21 questions, and is dated by Grant to around 1350.

The first two Questions are translated by Grant [10, pp. 131–135]. Question 1: “Concerning the book of Euclid, we inquire first about a certain statement by Campanus asserting that a magnitude decreases into infinity. First we inquire *whether a magnitude decreases into infinity according to proportional parts.*” Oresme refers to the edition of Euclid’s *Elements* by Campanus of Novara.

In his edition of the *Elements*, in Book I, after the common notions (*communium conceptiones*) and before the propositions, Campanus includes the following [3, pp. 58–59]:

Sciendum autem quod preter has communes scientias multas alias que numero sunt incomprehensibiles pretermisit Euclides quarum hec est una: Si due quantitates equales ad quamlibet tertiam eiusdem generis comparentur, simul erunt ambe illa tertia aut eque maiores aut eque minores aut simul equales. Item alia. Quanta est aliqua quantitas ad quamlibet aliam eiusdem generis, tantam esse quamlibet tertiam ad aliquam quartam eiusdem generis. In quantitatibus continuis: hoc universaliter verum est sive antecedentes maiores fuerint consequentibus suis sive minores. Magnitudo enim decrescit in infinitum. In numeris autem non sic, sed si fuerit primus submultiplex secundi, erit quilibet tertius eque submultiplex alicuius quarti quoniam numerus crescit in infinitum, sicut magnitudo in infinitum minuitur.

Given quantities a, b, c , there is a quantity d so that $a/b = c/d$, and if $b = na$ for some natural number $n > 1$, then $d = nc$, and the fact that there are quantities such that a/b is unbounded below is analogous to the fact the natural numbers are not bounded above.

Fix a magnitude a and a proportion r . We remove ra from the magnitude a , leaving $a - ra = (1 - r)a$. We remove $r(1 - r)a$ from what is left, leaving $(1 - r)a - r(1 - r)a = (1 - r)^2a$. We remove $r(1 - r)^2a$ from what is left, leaving $(1 - r)^2a - r(1 - r)^2a = (1 - r)^3a$, and so on. What is left is unbounded below, namely, “decreases into infinity”. The total of all the magnitudes removed is

$$ra + r(1 - r)a + r(1 - r)^2a + \dots ,$$

and this is equal to a . Indeed,

$$\sum_{n=0}^{\infty} r(1-r)^n a = ra \sum_{n=0}^{\infty} (1-r)^n = ra \cdot \frac{1}{1-(1-r)} = a.$$

Question 2: “Next we inquire *whether an addition to any magnitude could be made into infinity by proportional parts.*”

Oresme [10, p. 135]:

The third proposition is this: It is possible that an addition could be made, though not proportionally, to any quantity by ratios of lesser inequality, and yet the whole would become infinite; but if it were done proportionally, it would be finite, as was said. For example, let a one-foot quantity be assumed to which one-half of a foot is added during the first proportional part of an hour, then one-third of a foot in another [or next proportional part of an hour], then one-fourth [of a foot], then one-fifth, and so on into infinity following the series of [natural] numbers, I say that the whole would become infinite, which is proved as follows: There exist infinite parts of which any one will be greater than one-half foot and [therefore] the whole will be infinite. The antecedent is obvious, since $1/4$ and $1/3$ are greater than $1/2$; similarly [the sum of the parts] from $1/5$ to $1/8$ [is greater than $1/2$] and [also the sum of the parts] from $1/9$ to $1/16$, and so on into infinity.

Boyer [1, pp. 80–89] discusses infinite series in the work of Oresme.

Clagett [6]

Pedersen [23, p. 199]

Burton [2]

Kirschner [15]

Maieru [17]

Mazet [18]

Rommevaux [24]

Wallace [25, pp. 65–116, Chapter 3] on University of Paris

Grant [9]

Clagett [5]

Murdoch [21] and [20]

Gribaudo [12]

Juschkevitch [13, pp. 405–413]

REFERENCES

1. Carl B. Boyer, *The history of the calculus and its conceptual development*, Dover Publications, 1959.
2. Dan Burton (ed.), *Nicole Oresme’s De Visione Stellarum*, Medieval and Early Modern Science, vol. 7, Brill, Leiden, 2007.
3. H. L. L. Busard (ed.), *Campanus of Novara and Euclid’s Elements, volume I*, Franz Steiner Verlag, Stuttgart, 2005.
4. H. L. L. Busard (ed.), *Nicole Oresme, Questiones super geometriam Euclidis*, Franz Steiner Verlag, Stuttgart, 2010.
5. Marshall Clagett, *Some novel trends in the science of the fourteenth century*, Art, Science, and History in the Renaissance (Charles S. Singleton, ed.), Johns Hopkins Press, 1967, pp. 275–303.

6. ———, *Nicole Oresme and the medieval geometry of qualities and motions: A treatise on the uniformity and difformity of intensities known as Tractatus de configurationibus qualitatum et motuum*, University of Wisconsin Publications in Medieval Science, vol. 12, University of Wisconsin Press, 1968.
7. ———, *Nicole Oresme*, Dictionary of Scientific Biography, Volume 10: S. G. Navashin–W. Piso (Charles Coulston Gillispie, ed.), Charles Scribner's Sons, New York, 1970, pp. 223–230.
8. William J. Courtenay, *The early career of Nicole Oresme*, *Isis* **91** (2000), no. 3, 542–548.
9. Edward Grant (ed.), *Nicole Oresme*, *De proportionibus proportionum and Ad pauca respicientes*, University of Wisconsin Publications in Medieval Science, vol. 8, University of Wisconsin Press, 1966.
10. Edward Grant (ed.), *A source book in medieval science*, Harvard University Press, 1974.
11. ———, *God and reason in the Middle Ages*, Cambridge University Press, 2001.
12. Luciana Gribaudo, *La serie geometrica nell'opera quaestiones super geometriam euclidis di Nicole Oresme*, *Rend. Sem. Mat. Univ. e Politec. Torino* **35** (1976/77), 147–158.
13. A. P. Juschkewitsch, *Geschichte der Mathematik in Mittelalter*, Pfalz-Verlag, Basel, 1964, Translated from the Russian by Viktor Ziegler.
14. Anthony Kenny and Jan Pinborg, *Medieval philosophical literature*, The Cambridge History of Later Medieval Philosophy (Norman Kretzmann, Anthony Kenny, and Jan Pinborg, eds.), Cambridge University Press, 1982, pp. 9–42.
15. Stefan Kirschner (ed.), *Nicolaus Oresmes Kommentar zur Physik des Aristoteles*, Sudhoffs Archiv Beihefte, vol. 39, Franz Steiner Verlag, Stuttgart, 1997.
16. Brian Lawn, *The rise and decline of the scholastic 'quaestio disputata': with special emphasis on its use in the teaching of medicine and science*, E. J. Brill, 1993.
17. Luigi Maierù, *La teoria delle proporzioni nelle «Quaestiones super Geometriam Euclidis» di Nicole Oresme*, *Archives Internationales d'Histoire des Sciences* **40** (1990), no. 125, 258–277.
18. Edmond Mazet, *La théorie des séries de Nicole Oresme dans sa perspective aristotélicienne. 'Questions 1 et 2 sur la Géométrie d'Euclide'*, *Rev. Hist. Math.* **9** (2003), no. 1, 33–80.
19. John E. Murdoch, *Review of Nicole Oresme, Quaestiones super geometriam Euclidis, edited by H. L. L. Busard*, *Scripta Mathematica* **27** (1964), no. 1, 67–91.
20. ———, *Philosophy and the enterprise of science in the later Middle Ages*, The Interaction Between Science and Philosophy (Y. Elkana, ed.), Humanities Press, Atlantic Highlands, NJ, 1974, pp. 51–74.
21. ———, *From social into intellectual factors: An aspect of the unitary character of late medieval learning*, The Cultural Context of Medieval Learning (John Emery Murdoch and Edith Dudley Sylla, eds.), Boston Studies in the Philosophy of Science, vol. 26, D. Reidel Publishing Company, Dordrecht, 1975, pp. 271–348.
22. ———, *Infinity and continuity*, The Cambridge History of Later Medieval Philosophy (Norman Kretzmann, Anthony Kenny, and Jan Pinborg, eds.), Cambridge University Press, 1982, pp. 564–592.
23. Olaf Pedersen, *Early physics and astronomy: A historical introduction*, Cambridge University Press, 1993.
24. Sabine Rommevaux, *Un exemple de question mathématique au Moyen Âge*, *Ann. of Sci.* **63** (2006), no. 4, 425–445.
25. William A. Wallace, *Causality and scientific explanation, volume 1*, University of Michigan Press, 1972.

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