Midterm
Term: Fall 2016

Student ID Information

Last name: ___________________________  First name: ___________________________

Student ID #: _______________________

Course Code: MAT B44
Course Title: Differential Equations I
Instructor: Jordan Bell
Date of Test: October 25
Time Period: Start: 3:10 pm
End: 5 pm
Duration of Test: 1 hour 50 minutes
Number of Test Pages: 12 pages
(INCLUDING THIS COVER SHEET)
Additional Materials Allowed: Scientific calculator

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Grade Table
1. (5 points) Integral curves

The figure plots the integral curves for one of the following five ODE's. Explain why four of the ODE's can be excluded.

1. $y' = \sin(t) + y$
2. $y' = \frac{1}{t^2} + y$
3. $y' = -\sin(y)$
4. $y' = \sin(t)$
5. $y' = \frac{y}{t^2}$
2. (7 points) Euler’s method
   
   (a) (5 marks) For the initial value problem
   \[ y' = ty, \quad y(1) = 2, \]
   use Euler’s method with step size 0.1 to approximate \( y(1.2) \). Useful formulas:
   \[ y_{n+1} = y_n + hf(t_n, y_n), \quad y(t_n) \approx y_n. \]

   (b) (2 marks) Work out an explicit solution for this IVP.
3. (8 points) **Homogeneous first order equations**

(a) (2 marks) \( y' = F(y/x) \). Write this ODE in terms of \( v \) and \( x \) where \( v(x) = \frac{y(x)}{x} \).

(b) (6 marks) Solve the IVP

\[
y' = \frac{y}{x} + \frac{x}{y}, \quad y(1) = 1.
\]

Determine the domain of the solution.
4. (7 points) Exact equations

Find an explicit solution of the IVP

\[ y' = \frac{2y + 2x}{2x - 2y}, \quad y(0) = 1. \]

Determine the domain of the solution.
5. (6 points) Logistic ODE

Find an explicit solution of the IVP

\[ y' = y(1 - 3y), \quad y(1) = 1. \]

Useful formulas: Partial fractions

\[ \frac{1}{y(1-3y)} = \frac{A}{y} + \frac{B}{1-3y}. \]
6. (6 points) Picard iterates

Calculate the Picard iterates $\phi_0, \phi_1, \phi_2$ for the IVP

$$y' = \frac{t}{y}, \quad y(0) = 1.$$ 

Useful formulas:

$$\phi_{n+1}(t) = y_0 + \int_{t_0}^{t} f(s, \phi_n(s)) ds.$$
7. (8 points) Constant coefficient second order homogeneous equations

Find a solution of the IVP

\[ y'' + y' + y = 0, \quad y(0) = 0, \quad y'(0) = 1. \]
8. (8 points) Variation of parameters

Find a general solution of the inhomogeneous ODE

\[ y'' + 10y' + 9y = t. \]

Useful formulas:

\[ v'_1 = -\frac{y_2 g}{W}, \quad v'_2 = \frac{y_1 g}{W}. \]
9. (5 points) Difference equations

For $y_{n+1} = f(y_n)$,

$$y_n = f(y_{n-1}) = f(f(y_{n-2})) = \cdots = f^n(y_0).$$

For $f(x) = \rho x + b$ with $\rho > 1$, using $\sum_{k=0}^{N} r^k = \frac{1-r^{N+1}}{1-r}$, work out a formula for $y_n$. 
Page for work.
Page for work.