MAT B44 first order ODE example 9

Jordan Bell
Department of Mathematics, University of Toronto

September 13, 2016

\[(e^x \sin y + 3y) + (3x + e^x \cos y)y' = 0.\]
\[M(x, y) + N(x, y)y' = 0.\]
\[M(x, y) = e^x \sin y + 3y, \quad N(x, y) = 3x + e^x \cos y.\]
\[M_y = e^x \cos y + 3, \quad N_x = 3 + e^x \cos y.\]
\[M_y = N_x \text{ means equation is exact.}\]
\[F_x = M, \quad F_y = N. \quad F \text{ can be calculated using either. Using } F_x = M,\]
\[F(x, y) = e^x \sin y + 3xy + g(y).\]
\[F_y(x, y) = e^x \cos y + 3x + g'(y) \text{ and } F_y(x, y) = N = 3x + e^x \cos y, \text{ so } g'(y) = 0.\]

Take \(g = 0\), as for what we are doing so if we have a choice of constant we choose it to be 0,
\[F(x, y) = e^x \sin y + 3xy.\]

The equation \((e^x \sin y + 3y) + (3x + e^x \cos y)y' = 0\) is the same as \(M(x, y) + N(x, y)y' = 0\) which is the same as \(F_x + F_y y' = 0\) which is the same as
\[\frac{d}{dx}(F(x, y(x))) = 0.\]

Then
\[F(x, y(x)) = C.\]
\[e^x \sin y(x) + 3xy(x) = C.\]

This cannot be simply written as \(y(x)\) in terms of \(x\).
For IVP \(y(0) = y_0, \quad \sin y_0 = C.\)

So solution of IVP satisfies
\[e^x \sin y(x) + 3xy(x) = \sin y_0.\]